Dances and Affordances: The Relationship between Dance Training and Conceptual Problem-Solving

CHRISTIAN KRONSTED AND SHAUN GALLAGHER

Abstract. *It is often argued by educators and researchers that access to the arts leads to increased academic performance. However, it is not clear why such access does so. We here use autopoietic enactive embodied cognition and ecological psychology to explain the relationship between dance training and conceptual problem-solving. We investigate four features of dance training that are beneficial for conceptual problem-solving and critical thinking: empathy, affordance exploration, attention change, and habit breaking. In each case, we will see that the embodied sensorimotor skills developed through dance practice are a form of affordance exploration that can carry over into the realm of conceptual problem-solving. Hence, since some of the skills needed in conceptual problem-solving are the same ones developed and trained through dancing, when we train dance, we also train some of the relevant skills for conceptual problem-solving and critical thinking.*

A growing field of empirical research on the impact of arts education suggests that access to the arts has a positive impact on student performance. For example, it has recently been suggested that access to arts education (conceived broadly as music, visual arts, theater, and dance) enhances skills in empathy, creativity, and critical thinking.[1] Dance training, in particular, is said to increase empathy and creativity, as well as conceptual problem-solving skills.[2] Although the benefits and connections between dance and conceptual activities can be observed empirically, it is not theoretically clear why dance training can lead to improved conceptual problem-solving skills.

Christian Kronsted is a PhD student in philosophy and cognitive science at the University of Memphis and a former professional break dancer.

Shaun Gallagher is the Lillian and Morrie Moss Chair of Excellence in Philosophy at the University of Memphis and holds a secondary appointment as professorial fellow on the Faculty of Law, Humanities and the Arts, at the University of Wollongong (Australia). He is the recipient of the Humboldt Foundation's first Anneliese Maier Research Award (2012–18).

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We propose an enactivist and ecological affordance-based approach to explain the connection between conceptual problem-solving and dance. When looking at dance studies, phenomenology, and enactive embodied cognition, we find that dance can be understood as a form of affordance exploration. Specifically, dance training teaches the student to experiment with affordances and bodily possibilities using sensorimotor attunement. Dance training teaches the student to see new possibilities for action within themselves and in the environment by heightening the student's kinesthetic, proprioceptive, haptic, auditory, and other forms of perception. Further, dance training teaches the student various forms of attention: toward the environment, the body, and others. In this paper, we investigate four features of dance and improvisational dance that we believe are beneficial for conceptual problem-solving and critical thinking: empathy, affordance exploration, attention change, and habit breaking. In each case, we will see that the embodied sensorimotor skills developed through dance practice are a form of affordance exploration that can carry over into the realm of conceptual problem-solving. In short, some of the same skills needed in conceptual problem-solving are those developed and trained through dancing. The link to conceptual problem-solving is made through a growing body of research on the idea that symbol manipulation involves sensorimotor processes allowing the subject to engage with and act on affordances.

Theoretical Commitments—The Enactive Action-Affordance Loop

The theory of affordances posits perception as direct, contextual, and action-oriented. Perception is not an internal process taking place within the perceiver, but rather a product of direct relationships between perceiver and objects. An affordance is standardly defined as the *possibility for action* as it is directly perceived in an object.[3] To creatures like us, that is, with a particular bodily configuration that includes jointed limbs that bend, a chair affords sitting. Similarly, doorknobs afford door opening, rope affords pulling, and buttons afford pushing. However, this applies only to creatures who have specific kinds of bodies, who live specific kinds of lives, and who are enculturated in specific ways. Doors do not afford opening for lions or ants, and neither do buttons afford pushing. As Varela, Thompson, and Rosch explain, "Stated in precise terms, affordances consist in the opportunities for interaction that things in the environment possess relative to the sensorimotor capacities of the animal."[4] In this affordance-based way, we see the world in terms of what we can do with it.

We do not pay equal attention to the entire field of affordances available to us, however. Rather, depending on our current engagement with the world, some affordances become more or less salient. If the fire alarm goes

off on campus and I have to leave my office, the affordances to push buttons on the hallway soda machine are not particularly salient. On the other hand, if it is a hot Friday afternoon and I am taking a break outside my office, the soda machine not only becomes salient, but it solicits my actions. In other words, the field of affordances can be seen as "[t]he affordances that stand out as relevant for a particular individual in a particular situation; i.e., the multiplicity of affordances that solicit the individual."[5] Hence, while we stand in relation to innumerable affordances in our environment, only contextually salient affordances solicit us into action.

Although Gibson defines affordances relative to an individual, the recent conceptions of the "landscape of affordances"[6] and "affordance space"[7] include affordances defined relative to a form of life or a culture. Within a particular form of life, agents may develop more highly specific skill sets that lead to skilled action and highly specific affordances. Hence, the affordance space of the expert engineer is very different from the affordance space of the expert carpenter. Thus, activity, skill acquisition, and social practices lead to changes in affordance fields and in salience in a way that is important for understanding changes in attention. Furthermore, affordances and their salience are modulated by bodily affective states such as hunger, thirst, energy, happiness, and depression: "It's not just whether 'I can' or 'I can't' that modulates affordances, but also whether I have the energy, the interest, or the desire to engage in a particular action. Likewise, psychological changes bring along physical, affective, and social changes that modulate affordances."[8]

Following Gibson (and others), enactivism posits that cognition and action are closely intertwined. Furthermore, rather than being brain-centered, enactivism argues that perception and cognition are the achievement of the *entire organism* as it is dynamically coupled to the environment. In other words, for enactivists, it is a category mistake to point to just brain processes and say "there is the mind."[9] Rather, cognition is the ongoing achievement of autopoietic systems as they dynamically and holistically couple with environments.[10]

For enactivism, life and mind are continuous with one another.[11] Autopoiesis, the self-organizing process within which precarious systems maintain their continued existence, is the basic mechanism with which cognition begins. Living organisms, in order to maintain their existence, engage in constant physical self-adjustments to achieve desired metabolic states. For basic forms of animal cognition, as well as complex human cognition, autopoiesis is an ongoing process of moving through and responding to one's environment in order to stabilize metabolic conditions.[12] For autopoiesis to continue, organisms must attune themselves to their environment in a way that defines affordances for actions and that brings the organism into or closer to a desired state. In this process, not only does the environment

solicit action but the performing of such action leads to new affordances, so that, in movement, perception, and cognition, the agent's body and environment are intimately linked in ongoing feedback loops. The mind is not a passive receiver of input; rather, the organism brings forth its meaningful world through its own actions. Rather than thinking of vision, hearing, and other sensory modalities as passive, we should think of them as akin to haptic touch, which is active and exploratory. On the enactive paradigm, perception and cognition, in general, are active processes, as the organism constantly attempts to optimize its attunement to the environment.[13]

Enactivism thus understands "the mind" as situated, the result of brain, body, and environments being coupled together. What is salient to an organism through perception and in regard to action depends on its body, its emotions, its history, its long- and short-term projects, and an overarching enculturation process. Such contextualized agents will act differently depending on their situation and bring forth perceptual experiences highly dependent on that situation.

Cognition, from the most basic to the most advanced, happens in the dynamic unfolding of interactions with affordances in the *material environment*.[14] For example, if I want to sculpt a dragon out of clay, I act upon the material. Each of my actions will bring out new possibilities and limitations in the emerging form of the material. Malafouris's example of the potter at her wheel specifies this as an ongoing process that he likens to a dance: "This may allow us to understand the dynamic coupling between the potter and the task environment as a dance between equal partners, the potter leading the dance at some times and the potter's 'situation' leading it at other times."[15] Hence, as I am actively working with the material, a dynamic loop emerges between possibilities and actions. In my interaction with the material, my intentions are brought forth.[16] This notion becomes important later as we look to the methods of dance improvisation. Each time I act on an affordance in the material, the material changes shape and affords new possibilities for my action. This becomes a fluid process in which the material and the cognizer coconstitute one another as one system.

Whether we are sculpting, throwing a pot, or dancing, the material that we are engaging with presents us with different affordances for action. In other words, we see possibilities in the material and as we engage with the material. On this view, *cognition happens in the interaction between worldly material affordances and the agent*.

Enactivism, Social Cognition, and Participatory Sense-Making

Since we will be investigating dance, an inherently social enterprise, it is important to also have a look at what enactivism says about social cognition.

In terms of social cognition, enactivism emphasizes processes driven by embodied ecologically embedded interaction. It stands in contrast to classic social cognitive theories, which claim that our primary way of understanding others is effected by constructing a folk-psychological theory of what others might be thinking (theory theory) or by running imaginative simulations in which we put ourselves in the other's shoes (simulation theory). Rather, intersubjective meaning for human agents is created in the dynamic coupling and coordination between agents. These couplings and coordinations happen through a combination of cognitively cheap, partly automatic processes of embodied synchronization, and active autonomous efforts of the agent.[17] When human agents engage face to face in primary intersubjective interactions[18] and when, in pragmatic contexts, they coordinate with each other in secondary intersubjective interactions,[19] they synchronize by aligning basic processes such as speech patterns, posture, gaze, and breathing. "Coordination is typically easily achieved by simple mechanical means and, when cognitive systems are involved, it does not generally require any cognitively sophisticated skill. On the contrary, it is often hard to avoid," state De Jaegher and Di Paolo.[20] Humans have a naturally developed propensity to attune to one another. Gaze following, body posture, gesture, breathing, and other embodied generally subpersonal processes naturally synch up in social interactions and help to direct attention to the salient features of the situation.[21] De Jaegher and Di Paolo define social interaction in the following way:

> Social interaction is the regulated coupling between at least two autonomous agents, where the regulation is aimed at aspects of the coupling itself so that it constitutes an emergent autonomous organization in the domain of relational dynamics, without destroying in the process the autonomy of the agents involved (though the latter's scope can be augmented or reduced).[22]

Not unlike our sculpting and potter's examples (but with the addition of possible active reciprocity), action from one agent creates affordances for the other agent in what is often a fluently unfolding circle of affordance, action, and upkeep of the coupling. Simply standing in line together at the grocery store or (in most cases) walking past one another does not count as social interaction. Rather, agents must align (although often imperfectly) with one another so that each brings out new reactions in the other. This also depends on physical and social contexts and affordances. Walking past one another in a narrow hallway may call for interactive negotiations that may range from nonconscious to conscious adjustments. Similarly, each agent in socially coupled interactions may keep the engagement going by acting on social affordances provided by the other. On this model, conversation is the paradigm case. For a conversation to go well, the two speakers must take turns

speaking and listening. Active listening prompts the speaker to provide information; active speaking prompts the listener to interact and respond, both cases of "participatory sense-making" in which humans as dynamical systems coordinate to bring about new meanings above and beyond what each system would do on its own.[23] Meaning is created between participants as they interact. In each step of the process, the history of past coordination and the immediate history of the current coordination affect the dynamic unfolding of the current social coupling. The interaction itself forms a communicative system with its own organization perpetuated by each participant acting on social affordances and being acted on and, at the same time, going beyond what either participant can accomplish on her own.

Importantly here, understanding the other is not a matter of internally constructing a theory or simulation of her mental states. Rather, the understanding is brought about through the interaction with the other, in communication, in joint actions, in instituted practices, and so on. The interactive coupling between agents bring about a *direct understanding* of the other as an emergent property of interaction. We do not first perceive the other's action as mere movements that lack intrinsic intentionality and then make inferences to a meaning hidden behind them. Rather,

> [w]e enactively perceive the actions and emotional expressions of others as forms of intentionality—i.e., as meaningful and directed. Enactive perception of others means that we see their emotional expressions and contextualized actions as meaningful in terms of how we might respond to or interact with them. Others present us with social affordances. Accordingly, our understanding of others is pragmatic and it references their actions in context.[24]

Importantly for enactivism, interaction itself gives us access to the intentions and emotions of the other as well as to a field of shared interactively created meaning. For example, when dancing, we do not build a theory of what the other person wants us to do (as if, when dancing, I am operating on rules such as "When she pushes, I step backward, and when she is done pushing, it is my turn and then I push forward"). Rather understanding the intention of the other happens as my partner grabs my hand and gently pushes backward, which makes me take a step backward. Through the push of the other, their physical movement, interaction is made, and that interaction just is the way that I understand the other's intention, which itself is being shaped in the interaction. Thinking does not happen first, followed by action. Rather, to understand that the other person wants me to take one step backward as she steps forward is just to take one step backward as she steps forward. As we saw, cognition is constituted by the dynamic interactions between brain-body-environment couplings. Similarly, understanding others is not primarily something we "get right" as an intellectual practice; understanding others is something we do.

Dance as Affordance Exploration

With this underlying framework in place, we can begin to investigate how dance training can contribute to the participant's skills in conceptual problem-solving. To do so, we will be looking at the phenomenology of dance and its relationship to affordances. We will see that dance training teaches its participants to explore and generate new affordances through four bodily practices: empathy, affordance exploration, attention change, and habit breaking. In turn, these practices learned through dance overlap with the set of embodied practices used when doing mathematics, critical thinking, or other conceptual problem-solving tasks.

Professional dancers who also engage in academic reflection have over the years produced various accounts of cognition and phenomenology during rehearsed and improvisational dance. Across this literature, it is commonly agreed that dance as praxis involves special modes of sensing, cognizing, and moving, in which those three modes are not distinct but rather take place together in a continuous loop.

In *The Primacy of Movement*, Sheets-Johnstone describes improvisational dance as an active exploration of one's own possibilities within the environment. Improvisation is an ongoing process of creation in which the dancer actively creates shape, form, and force while simultaneously perceiving and investigating those shapes, forms, and forces. Improvisation on this account is a playful engagement with affordances found in music, environment, and the ever-changing form of one's own body:

> To say that in improvising, I am in the process of creating the dance out of the possibilities that are mine at any moment of the dance is to say that I am exploring the world in movement; that is, at the same time that I am moving, I am taking into account the world as it exists for me here and now in this ongoing, ever-expanding present.[25]

For each moment in improvisation, the dancer reacts to her own current kinesthetic unfolding in order to make the next movement. That is, the body both creates affordances for itself and explores those affordances. The dancer is exploring by seeing where they can go next within the possibility space, and, for each exploration, a new possibility space is created.

Michelle Merritt builds on Sheets-Johnstone's account to argue for an enactivist account of cognition. In describing the cognitive loop in improvisation, Merritt writes,

> In a non-choreographed dance, it most certainly cannot be the case that the dancers are carrying out movement in the first manner characterized above. That is, they do not think before they act; they simply begin moving. There might be some pre-specified rules to the improvisation—you must maintain contact with a person or an object, e.g.,—but for the most part, movement is spontaneous and unplanned.[26]

On such accounts, the dancer is freely exploring herself and her environment as the movement discloses the world and the body. Movement in this regard is meaningful and intelligent; it's a form of sense-making. In an interactive coupling between agent, music, and environment, the dancer acts and reacts so that movement and thoughtful engagement with the world become *interchangeable*, Merritt argues:

> Movement just is thought, and thought, in the case of improvisational dance, consists in the movement. Furthermore, movement is not simply reactive; nor is it reflectively thought out ahead of time. It is, rather, dynamic, ever-shifting, and responsive to context. This dynamism—because it is so intelligent in its responsiveness—seems to require some sort of agent to whom the movement means something. In other words, it would seem wrong to insist that the movement is non-conscious or merely a biological maintaining of the organism below the conscious radar. The movement means something to the persons enacting it.[27]

As Merritt points out, improvisational dance is not mindless; rather, it is an ongoing mindful process. It involves exactly the type of attuned focus that allows the dancer to do what they do. Vida Midgelow, in her chapter in *Back to the Dance Itself*, echoes the point made by Merritt and argues for improvisation as a deliberately conscious process. Rather than being "unplanned, unpracticed, make-do activity as is perhaps a common misconception," improvisational dance is based in rehearsal (practice) and the mastery of skilled *modes* of moving.[28] As Midgelow goes on to argue, "[I]n these modes, improvisation is generally a rigorous, focused, and purposeful way of going about things."[29] In this regard, improvisation is a highly attentive activity. Improvisational dance on this account consists of different modes of sense-making movements dictated by the specific dance form, music, audience, mood, and other constraints. The dancer's "freedom" is tightly constrained by the various modes of movement that have been perfected in rehearsal. Thus, while there are no set steps or preset movements for dance improvisers, there are modes of moving and classes of steps that are appropriate to what is being afforded by the environment.

Improvisational dance involves an ongoing awareness of where the body has just been, where the body is, and where the body is going through an engagement with affordances. For example, Ann Albright describes dance improvisation as an active form of feeling—feeling as a verb, not a noun.[30] For Albright, improvisers train to pay acute attention to kinesthesia, proprioception, and haptic sensation, so that they can dynamically couple themselves with their environments and/or their dance partner(s) in a controlled fashion. Through heightened awareness of these modalities, dancers can create ongoing coherent movements with their partners by feeling and responding in an ongoing loop. As the dancer's movements unfold, new

sensory affordances are created in the moment which carries the interaction forward. Hence like the enactive feedback loop of participatory sensemaking discussed earlier, dance improvisation involves an ongoing cycle of acting on affordances and being acted upon. Each movement brings out new possibilities that the dancer must engage to keep the participatory sensemaking activity going.

Dance and Understanding Others

With this understanding of dance improvisation as a practice, we can begin to see the implications for conceptual problem-solving and critical thinking. In this section, we demonstrate how dance training can give the student an increased understanding of the actions and motivations of other agents. In the next section, we show how dance can increase the student's skill in symbol manipulation. In both cases, it is a story about affordances. We begin with the other-directness of dance and its ability to teach empathic perspective taking.

In going through the movement-affordance loops described above, the dancer learns how different kinesthetic and proprioceptive configurations of her body and the body of her partner(s) provide new possibilities for an ongoing engagement—that is to say, new intersubjective affordances. This requires a heightened awareness not only of one's own body and the bodies of others but also an awareness of how others perceive and attune to one's own body. In studying reflective practices in and around dance, Tembrioti found that students taking dance classes developed a heightened awareness of how their own bodily expressions were seen from the perspective of others.[31] Students who were asked to reflect upon their experiences of creating dance choreographies showed an increased awareness of how others might perceive various forms of expression. Hence, when dance students train to think about culturally contextualized audience responses, they are training empathy. This way of seeing oneself through the gaze of the other is an important component in critical thinking and conceptual problem-solving since such tasks often involve having to take the perspective of the other.[32] There has been a long-standing agreement about the link between empathy and critical thinking in both education theory and empirical educational studies.[33] The general notion in this literature is that increased empathy provides the empathizer with a better understanding of the actions and justifications of others. This is an advantage in the context of joint problem-solving, for example. Hence, the expressive other-directed nature of dancing supports the empathetic component of critical thinking.

A more subtle point regarding dance, empathy, and critical thinking has to do with the increased understanding of the experiences of others gained from acquiring higher degrees of movement expertise. The *broad* movement

expertise gained through dance training allows the dancer to resonate better with the actions, feelings, and experiences of other people as they perform similar or related movements. Empathy is not just a higher-order cognitive act based, for example, on conscious imagination or narrative practices[34] but also a "basic" embodied experience of the other through various resonance processes.[35] Dancers engaged in live dance performance, like audiences who are perceiving such performances, are subject to contagion of affect, basic empathy, and the alignment of low-level embodied processes such as breathing and kinesthesia.[36] Rather than a purely intellectual judgment, even just the perceiving of improvisational performance is a fully embodied experience encompassing affect, proprioception, kinesthesia, and other sense modalities.[37] Thus, when we perceive other people move, we register those movements across a number of highly embodied sense modalities. According to Bresnahan, training dance allows the subject to have a more in-depth perceptual (and motoric) understanding of the motions of others due to an increased affective and kinesthetic understanding that comes from having trained similar motions oneself.[38] This conclusion is further supported by neuroscience research that shows that, for trained dancers, the same neurons will fire when performing a dance as when observing others perform a dance.[39] Specifically, researchers found greater bilateral activations in premotor cortical areas when expert dancers observed movements they had been trained to perform compared to movements they had not.[40] This is explained in terms of the mirror neuron system that integrates observed actions of others with an individual's personal motor repertoire.

The conclusion that dance training allows us to understand the perspective of the other is further supported when we think deeper about what improvisation allows students to experiment with—namely, a kind of semiotic manipulation.[41] Through careful analysis, Deans observed that children learn a form of metaphoric understanding through dance.[42] Using improvisation to experiment with force, flow, spatiality, time, and gesture, students learn how various forms of movement denote certain broad meanings within their culture. They engage with "enactive metaphors" and gain expertise in how to communicate with them.[43] In dance they enact metaphors, putting them into action or bringing them into existence through movement. Students learn a highly culturally determined vocabulary of bodily messages by experimenting with the reception of expressive movements. Further, children who first went through dance training and later took poetry workshops reported using similar strategies when creating poetry as when creating dances. More specifically, both dancing and poetry involve an attunement to how others might receive one's expression within a metaphoric framework.[44] In this regard, dance improvisation (for children and adults alike) trains the participant in embodied communication, which requires attunement to and understanding of the context of the other's

action. This also means that, through experimentation and improvisation, dancers learn what social affordances they can create for others to interact with. This understanding is generated through the improvisational interactions with those others in the ongoing dance engagement. Learning to dance and improvise with others is to gain a form of embodied understanding of others that can then be utilized in the realm of conceptual problem-solving.

The important point here is that an expanded movement vocabulary and expertise and a better sense of how a wide range of movements are experienced scaffold the dancer's ability to understand others and to think critically about their own and others' movements and actions. In this respect, we have highlighted several overarching points regarding the relations between dance, empathy, and critical thinking. Empathy is an important component of critical thinking, and dancing helps teach the student how to take the perspective of other people. Gaining an enhanced movement vocabulary and fluency increases the dancer's understanding of what others experience as they produce similar movements in the world. The dancer thus learns how her movements produce social affordances with which other persons can interact.

Sensory-Motor Exploration and Symbol Manipulation

So far so good, but math symbols do not require taking the perspective of the other, and neither do a lot of other conceptual-thinking tasks. Dance training, however, through its emphasis on sensory openness, attention, and habit-breaking, is also a form of affordance exploration that carries over into practices that involve symbol manipulation. The core concept here is that both dancing and symbol manipulation involve sensorimotor engagement with affordances and, therefore, have overlapping skill sets.

We can begin this discussion by looking to research in neuroscience that indicates that areas of the brain typically associated with symbol manipulation and abstract language use are also activated during dancing.[45] That is to say the same mechanisms in use for symbol manipulation and abstract thought are also utilized during dance.[46] Hence, looking at brain activation provides a clue that supports an initial observation that dancing and conceptual problem-solving are connected through overlapping mechanisms in the brain. By strengthening our competence in dance, we also strengthen the same neuronal mechanisms in play when doing math and symbol manipulation (on increased neuroplasticity in dancers).[47] However, we can fine-tune our analysis by looking at research on math as an affordance-based sensorimotor activity.

A growing body of empirical research demonstrates that symbol manipulation, such as the manipulation of mathematic equations, is a matter of affordance manipulation.[48] That is to say, we experience and manipulate

mathematical objects in a way similar to our manipulation of physical objects. Lakoff and Núñez, for example, argue that mathematical operations rely on basic body-related activities, which include "basic spatial relations, groupings, small quantities, motions, distributions of things in space, changes, bodily orientations, basic manipulations of objects (e.g., rotating and stretching), iterated actions, and so on."[49] Such activities involve sensorimotor interactions with affordances. The claim, then, is that many of the skills and techniques used to manipulate affordances in dance are similar to skills and techniques used when manipulating affordances in the mathematical realm; in this regard, dance training can aid in strengthening the dancer's conceptual and symbol problem-solving skills. We need to unpack this claim in more detail, however.

Consider doing math on paper or on a white- or blackboard. This is a process of manipulating objects; we move numbers from one side of the equation to the other, cross out numbers, move down lines of operation, look at the board, gesture at parts of equations, move around the board, try out different lines of thought by physically carrying out the mathematical operations line by line. As Anderson points out, these various movements and actions done around mathematical problem-solving are not epiphenomenal to some internal process.[50] Rather, gesturing, moving, acting, and manipulating external objects are part and parcel of the process of conceptual problem-solving. The "mathematical thoughts" are not produced in the brain and then externalized. Rather, the coupling between the symbols on the board and the agent is the cognitive process (here it can be helpful to think back to our previous example regarding working with clay). The point is that the cognitive process of symbol manipulation is the physical act of a body manipulating real objects in the world—in this case, numbers and operators on a board or on paper. The numbers and mathematical operators afford the agent various possibilities with which they engage through an unfolding loop; as I manipulate equations new possibilities for action emerge, which allow for further engagement. We can allow for the idea that these processes may in some cases be internalized in the sense that we can perform such operations in our imaginations, without physically moving. But such imaginative manipulations are derivative, as we know from how we go about learning math.

The empirical literature supports the conclusion that symbol manipulation is an embodied endeavor. For example, math comprehension is reduced when subjects are not allowed to use gestures.[51] Additionally, using actual manipulative objects to teach math has been found effective for immediate learning growth in student populations suffering from math-learning disability.[52] Research on virtual manipulative and their affordances in mathematics-learning apps demonstrate that student learning becomes more efficient when students interact with helping affordances within the app. For

example, the learning progress of preschool, K, and second-grade students was significantly improved when engaging with physically manipulative affordances within math-learning apps.[53] Apps that involve dragging and manipulating virtual blocks around the screen can enhance student learning by making the correct blocks easier to manipulate and the wrong blocks slightly harder to manipulate. Virtual manipulatives were also found significant for learning outcomes when university students were using virtual manipulatives to solve advanced geometry problems.[54]

In addition, Landy and Goldstone found that manipulation of the spatial properties of an image equates to manipulating its mathematical content; when manipulating the distance between numbers and mathematical operators the subject's math comprehension was diminished.[55] Just as chairs, baseballs, and hammers reflect affordances, so do equations. Hence, by manipulating various spatial features of the equations, Landy and Goldstone were able to have even expert mathematicians perform invalid mathematical operations on equations. Additionally, to show that symbol manipulation is a sensorimotor skill, in which one understands the physical constraints of the equation, Landy and Goldstone had participants solve equations on a screen against a moving background.[56] The movements of the background were either moving with or against the direction in which subjects were supposed to move symbols in order to solve the equation. They found that participants had difficulty solving equations when the background was moving incongruently to the direction needed to solve the equation but less difficulty when movements were congruent. Even the sense of the salience of different mathematical operators is given to us in terms of spatial affordances. For example, Landy and Linkenauger found that, when placing an object underneath the multiplication sign and under the addition sign, most participants thought that the object underneath the multiplication sign was spatially closer to them.[57] Through training, we learn that, in the order of operations (PEMDAS—which stands for parenthesis, exponents, multiplication, division, addition, subtraction), multiplication is done before addition; hence, the multiplication sign appears more salient and physically closer to the trained mathematician. As Anderson puts the point,

> A natural extension of the view that equations have spatial affordances is that they invite us to act *on and with* them, manipulating and moving their parts around as part and parcel of the problem-solving process. In this view, to learn algebra is to acquire a sensorimotor skill, and acting in accord with the rules of algebra is a matter of learning to see and act in accordance with the transformations that the equations afford.[58]

We see, then, that, both in dancing and in processes of symbol manipulation, agents are exercising embodied sensorimotor skills, moving through affordance-action loops as they engage with the relevant affordances. Both

dancing and symbol manipulation are matters of acting on affordances and attuning one's body to an ongoing coupling with (physical, social, or symbolic) environments. Whether we are engaging with shaping clay, dancing Bachata, or manipulating complex math equations, we engage in sensorimotor loops of acting on affordances. As we continue to act, new possibilities arise that require further action and attunement. The symbols themselves constrain and afford action that the agent exploits. Thus, bettering our affordance-manipulation competency through dance also improves our affordance-manipulation skills during problem-solving processes of symbol manipulation.

Attention, Change, and Habit Breaking

Dancing, symbol manipulation, and conceptual problem-solving are further connected with regard to attention. Through dance training, students can learn to shift their attention from conventionally salient features of objects to novel features and new affordances associated with those objects. This allows the dancer to pursue new lines of engagement with the object, a skill that is both transferable and useful when stuck during conceptual problem-solving. Doughty provides a description of dance improvisation as a mode of perception in which the dancer becomes hyperaware of both her body and how the body is being affected by the environment.[59] Doughty describes the goal of this mode of sensing as one in which the dancer approaches objects in the environment as novel rather than well known. The goal of this kind of improvisation is to get into a state of mind in which every encounter with objects is treated as a "new" encounter so that the dancer is forced to experiment with the affordances of the object.[60] Thus, improvisation is here described as a form of creative affordance exploration that emerges from treating objects "as if" they are encountered for the first time. A classic example of this can be found in dance exercises in which the improviser is asked to "dance with" an object such as a box or a table. Here the improviser must explore the affordances of the object through various lines of engagement. Through this unfolding, the improviser discovers novel possibilities within the object and herself. We see, then, that, in dance improvisation, the dancer playfully practices shifting her attention for the sake of exploration. Dance improvisation thus involves actively trying to bring out new affordances in the engagement between one's body and objects by shifting one's attention toward features of the object that are normally nonsalient to oneself.

We can compare this form of dance improvisation to the process of trial and error in problem-solving. When stuck on a problem, whether symbolic or conceptual, solving the problem often involves shifting one's attention to different features of the symbolic structure or problem and then pursuing new lines of engagement from that newly found starting point. Trial and

error problem-solving is often a matter of seeing new affordances and then pursuing a line of engagement from those affordances. For example, when working on math equations the solution will often flow from starting the problem in a new way. Thus, the connection between dance improvisation and attention shifting should begin to become clear; the process of shifting one's attention to pursue new lines of engagement is the same whether one is dancing or solving math problems. The dance skill of shifting one's attention to previously nonsalient affordances in objects and environment can thus be utilized within the domain of conceptual problem-solving. Attention shifting teaches the dancer to follow novel paths of affordances unfolding within sets of constraints. Here it does not matter if the constraints and affordances are conceptual and/or symbolic versus physical. Shifting one's attention through embodied engagement with object affordances is the same skill either way.

The exploration of affordances can further be promoted through dance training by the practice of actively breaking movement habits. Manning describes how the dance improviser experiences time during improvisation in terms of space, affect, and affordances.[61] During improvisation, every moment is experienced as a new encounter with space, affect, and affordances; in such a moment, the body is tuned to respond through *habitual* action. Building on Manning, McDowall describes what she calls "the gap" as the body's desire in the moment to engage affordances in a habitual manner. The gap depends on the sensation of the body being solicited to act on an affordance with a habitual movement.[62] However, the gap is created when the agent resists the solicitation (the pull if you will) of the affordance, by acting on the affordance in a *nonhabitual manner*. The point here is that human cognition and action are highly habitual; when we act nonhabitually in a habitual situation, it is experienced phenomenologically as a felt resistance or an opening of tension between what I usually do and what I am doing. As an example, we invite the reader simply to think of any habitual behavior she might have tried to get rid of in the past. Part of the difficulty of breaking a habit stems from the body's desire to respond habitually to a highly soliciting affordance.

Through training, dancers can learn to become more cognizant of their own movement habits as they learn to pay attention to the experience of "the gap." By leaving "the gap" open—that is, to engage continually with affordances in a nonhabitual manner—novel forms of movement can be produced. Thus, as a technique to create new movements, dancers will pay attention to their body's pull toward habitual engagement and actively act against this pull. This way of actively moving against the habitual desires of the body can, in turn, open new possibilities for the improviser as she actively explores the affordances that emerge from moving against her own habits.

We can now see how habit breaking in dance relates to conceptual problem-solving. The habit-breaking mode of dance improvisation teaches the dancer to actively be aware of her own habits. Often being stuck on a conceptual problem involves repeating the same cognitive patterns: symbol manipulation and conceptual problem-solving are not activities outside the grip of habit. Thus, thinking outside the box often requires having to break the habitual pull of what the environment is affording. Hence, the dancer who becomes sensitive to and aware of her own cognitive habits and how to break them can, in turn, apply this skill to conceptual problem-solving. This may begin to explain why high school students who undertook afterschool dance training scored better on standardized creativity tests than nondance students.[63]

The idea of habit breaking can at first sound like a high-level sophisticated skill developed through cognizant and guided training above and beyond what an afterschool dance program might teach. However, much dance training teaches students to habit break without the student explicitly understanding the theoretical and phenomenological underpinnings of habit breaking. Dance students generally gain an awareness of their own sensorimotor habits and how to move against those habits, through even basic improvisation training. Dance teachers will often look at the posture, movement style, and step choices of their students and guide the student to break those habits. It is a general practice and important component in many improvisational dances (salsa, hip-hop, break dancing, house, Chicago footwork, just to mention a few) to understand one's own "go-to steps" and move away from those in order to increase improvisational creativity. Thus, the skill of habit breaking is learned both in the case of the cognizant professional dancer and the novice simply learning many dances that are improvisational in their nature. Overlap between habit breaking and conceptual problem-solving happens as students explicitly or implicitly use the techniques of habit breaking to pursue new lines of engagement during symbol manipulation or conceptual problem-solving. We often fall into habitual patterns of thinking and habitual patterns of problem-solving. Thus, the ability to recognize one's own habitual patterns becomes an important skill when we find ourselves stuck on a conceptual task.

Conclusion

Looking at the connection between dance and critical thinking, we have found that dance can improve critical thinking skills across four measures: empathy, affordance exploration, attention change, and habit breaking. Empathy is an important component in critical thinking, and dancing helps improve the dancer's perspective-changing capabilities. Improvisational dance teaches the dancer to shift her attention toward new and less

immediately salient affordances in the environment and teaches a mode of affordance exploration that can be carried over into the realm of conceptual problem-solving. Finally, dance training makes the student aware of his own movement habits and cognitive habits while emphasizing ways to break those habits. This is key since habit-breaking is an important skill for solving complex conceptual problems.

While we here have been able to draw some conclusions regarding dance and conceptual problem-solving by combining enactivist theory and empirical research from various fields, more research is still needed. Since we have shown that dance can help with conceptual problem-solving, it is natural to ask whether things also move in the other direction, that is, whether training in math and conceptual problem-solving makes better dancers. In other words, is the dance-symbol manipulation relation a two-way or a one-way relation? We think it is improbable that anyone could become better at mathematics from practicing dance without also practicing math or become a better dancer by simply practicing math. Thus, we have looked at the slim area in which dance and problem-solving overlap, namely, in the use of sensorimotor skills and affordance exploration. What we have suggested so far is that dance can help improve one's critical thinking and symbol manipulation skills regardless of one's initial starting point. We think that more research is also needed to explore the impact of dance on social cognition. Finally, it is worth asking what other forms of learning dance can contribute to. For example, what is the relationship between dance and perceptual learning or dance and practical skill? These are, of course, only a few of the questions that need to be asked when considering dance in relation to embodied cognition and learning.

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